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# Mathematical Analysis AB

## Finalità

The course provides the basic notion of calculus

## Programma

Number systems.

Rational and real numbers. Dedekind's completeness axiom. Least upper bound and greatest lower bound of sets. The archimedean property of integers. Density of rational numbers. Induction principle, Bernoulli's inequality and Newton's binomial formula. Combinatorics: permutations, dispositions and simple combinations. Complex numbers. Algebraic and geometric representation of complex numbers. De Moivre's formula. Roots of complex numbers. The fundamental theorem of algebra.

Numerical sequences and series.

Convergent and divergent sequences. Algebraic and order related properties of convergent and divergent sequences. Monotone sequences. Nepero's number. Some special sequences.

Convergent and divergent series. Necessary condition for convergence. Series with nonnegative terms: comparison, root and ratio tests. Leibnitz's test for alternating series. Series with arbitrary terms: convergence and absolute convergence.

Limits and continuity for functions of one real variable.

Limits of functions of one real variable: finite and infinite limits, limits at infinity. Limits along sequences. Algebraic and order related properties of limits. Limits of monotone functions.

Some fundamental limits of functions. Continuous functions. Algebraic and order related properties of continuous functions. Composition of continuous functions. Continuity of inverse functions. Continuity of elementary functions. The intermediate value theorem and its consequences. Weierstrass' theorem.

Differentiation of functions of one real variable.

The derivative of a real function of one variable and its geometrical meaning. Differentiability implies continuity. The algebra of derivatives. The chain rule and the inverse function theorem. Derivatives of elementary functions. Theorems by Fermat, Rolle and Lagrange and their consequences. Higher order derivatives. Taylor's formula. Lagrange's form of the remainder. Maxima and minima of differentiable functions. Convex functions and their properties. Antiderivatives. Integration by parts and the substitution method. Antiderivatives of rational functions (with degree two denominators only).

The Riemann integral.

The Riemann integral of a bounded real function of one variable and its geometrical meaning.

Properties of the integral. Integrability of monotone and continuous functions. The mean value theorem for integrals. The fundamental theorem of calculus and its consequences. Generalized integrals. Comparison between generalized integrals and series.

Ordinary differential equations.

First order ordinary differential equations. Solutions of first order linear differential equations. Bernoulli's equation. separable differential equations.

Second order linear differential equations with constant coefficients. Fundamental systems of solutions. Wronskian matrix. Lagrange's method of variation of constants.

## Attività d'esercitazione

A 40 hours long, exercise course will take place. Students are supposed to work out weekly assignments.

## Modalità d'esame

A final written exam takes place.

## Testi consigliati

E. Acerbi - G. Buttazzo: *Analisi matematica ABC vol.1*, Pitagora, Bologna, 2003

D. Mucci: *Analisi matematica. Esercizi/1. Funzioni di una variabile*

Pitagora, Bologna, 2004